СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
Дубна

E2 - 4955

B.M. Barbashov, S.P. Kuleshov,
V.N. Pervushin, A.N. Sissakian

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

EIKONAL APPROXIMATION FOR THE INELASTIC SCATTERING AMPLITUDE IN QUANTUM FIELD THEORY MODEL

1970

Барбашов Б.М., Кулешов С.П., Первушин В.Н., Сисакян А.Н. E2-4955

Эйкональное приближение амплитуды неупругого рассеяния в модели квантовой теории поля

В статье рассмотрены процессы рассеяния двух скалярных нуклонов с образованием скалярных мезонов в модели $\mathcal{L}_{вз.} = g:\psi^2(x)\phi(x)$. Случай рождения одной частицы исследуется детально. Показано, что в некоторой области импульсов рожденных мезонов неупругую амплитуду можно выразить через упругую, причём последняя представима в эйкональной форме.

Сообщения Объединенного института ядерных исследований

Дубна, 1970

Barbashov B.M., Kuleshov S.P., Pervushin V.N., Sissakian A.N. E2-4955

Eikonal Approximation for the Inelastic Scattering Amplitude in Quantum Field Theory Model

Processes of scattering of two scalar nucleons with production of scalar mesons in the model $\mathcal{L}_{int} = g:\psi^2(x)\phi(x):$ are considered. The case of single particle production is investigated in detail. It is shown that it is possible to represent the inelastic scattering amplitude in terms of the eikonal elastic amplitude in a certain region of the momenta of the produced particle.

Communications of the Joint Institute for Nuclear Research.

Dubna, 1970

1. Introduction

At present the eikonal representation for the elastic scattering amplitude attracts a great attention by that it can be successfully used for the description of a large number of experiments on high energy particle scattering.

As far as this representation has arised in the framework of non-relativistic quantum mechanics[1,2] a special basis of the eikonal or Glauber form of the scattering amplitude in the relativistic region was required. Among papers devoted to this problem we should mention the ones[3,4] in which the validity of the eikonal representation has been proved in the lowest perturbation orders. In recent papers[5,6] using the Logunov-Tavkhelidze[7] quasipotential equation the authors have obtained a Glauber representation for high energy small-angle hadron scattering under the condition that the local quasipotential is smooth.

A great number of articles[8-11] has been devoted to the study of the eikonal approximation in quantum field theory models.

In particular, in ref.[9] the eikonal representation for the elastic scattering amplitude has been derived in the scalar model $\mathcal{L}_{int} = g : \psi^2(x)\phi(x):$ by the functional integration method. The account has been made of the s-channel ladder and cross-ladder graphs without radiation corrections and nucleon closed loops, under the assumption of asymptotically high energies $s \to \infty$ and fixed momentum transfers t.

It is of great interest to explore whether it is possible to represent the inelastic scattering amplitudes in the eikonal form in the domain $\frac{t}{s} \ll 1$.

In the present paper we consider the processes of scalar nucleon scattering with meson production. The case of single particle production is studied in detail.

In a certain region of the produced particle momenta it turns out to be possible to express the inelastic scattering amplitude in terms of the elastic scattering amplitude in the eikonal form.

2. Inelastic Scattering Amplitude

The two-nucleon scattering amplitude with n -meson production can be written in the form /12/

$$(2\pi)^4 \delta(p_1+p_2-q_1-q_2-\sum_{i=1}^{n} k_i) f(p_1 p_2 | q_1 q_2 | k_1 \dots k_n) = \tag{2.1}$$
$$= \prod_{j=1}^{n} \frac{\delta}{\delta\phi(k_j)} F(p_1 p_2 | q_1 q_2 | \phi, \phi) \Big|_{\phi=0} ,$$

where the generating functional is expressed in terms of single-particle Green functions of a nucleon in the external field $G(p\,q|\phi)$ as follows

$$F(p_1 p_2 | q_1 q_2 | \phi_1, \phi_2) = \lim_{(p_\ell^2, q_\ell^2) \to m^2} \prod_{\ell=1}^{2} (p_\ell^2 - m^2)(q_\ell^2 - m^2) \tag{2.2}$$
$$\exp\{ i \int dy_1 dy_2 \frac{\delta}{\delta\phi_1(y_1)} D(y_1 - y_2) \frac{\delta}{\delta\phi_2(y_2)} \} \{ G(p_1 q_1 | \phi_1) G(p_2 q_2 | \phi_2) \}$$

The Green function

$$G(p\,q|\phi) = \int dx\, dy\, e^{-ipx + iqy} G(xy|\phi) \tag{2.3}$$

is found from the Klein-Gordon equation in the external field

$$(\Box - m^2 + g\phi)G(xy\,|\,\phi) = -\delta^4(x-y) \tag{2.4}$$

and can be written as a functional integral [13]

$$G(pq\,|\,\phi) = \lim_{p^2\to m^2} \frac{1}{p^2-m^2} \int dx\, e^{i(p-q)x} \int [\delta^4\nu]_0^{\infty} \tag{2.5}$$

$$\exp\{ig\int_0^{\infty}\phi[x+2p\xi+2\int_0^{\xi}\nu(\eta)\,d\eta]\,d\xi\} \;,$$

where

$$[\delta^4\nu]_{\tau_1}^{\tau_2} = \frac{\delta^4\nu\, e^{-i\int_{\tau_1}^{\tau_2}\nu^2(\eta)\,d\eta}}{\int\delta^4\nu\; e^{-i\int_{\tau_1}^{\tau_2}\nu^2(\eta)\,d\eta}} \;. \tag{2.6}$$

Inserting (2.5) into (2.2) and excluding the terms corresponding to unbound graphs and going to the mass shell, as has been done in ref. [9] we get

$$F(p_1p_2|\,q_1q_2\,|\phi_1,\phi_2) = \int dx_1\,dx_2\, e^{i(p_1-q_1)x_1+i(p_2-q_2)x_2} D(x_1-x_2) =$$

$$\int[\delta^4\nu_1]_{-\infty}^{\infty}[\delta^4\nu_2]_{-\infty}^{\infty}\exp\{ig\int_{-\infty}^{\infty}d\xi_1\,\phi_1[x_1+a_1(\xi_1)+\int_0^{\xi_1}\nu_1(\eta)\,d\eta]+ \tag{2.7}$$

$$+\,ig\int_{-\infty}^{\infty}d\xi_2\,\phi_2\,[x_2+a_2(\xi_2)+\int_0^{\xi_2}\nu_2(\eta)\,d\eta]\}\int_0^1 d\lambda\,\exp\{i\lambda\chi\}$$

where

$$\chi = g^2\int_{-\infty}^{\infty}d\xi_1\,d\xi_2\,D[x_1-x_2+a_1(\xi_1)-a_2(\xi_2)+\int_0^{\xi_1}\nu_1(\eta)\,d\eta-\int_0^{\xi_2}\nu_2(\eta)\,d\eta] \tag{2.8}$$

$$a_i(\xi_i) = 2p_i\,\xi_i\,\theta(\xi_i)+2q_i\,\xi_i\,\theta(-\xi_i) \;. \tag{2.9}$$

We consider, for example, the two-nucleon scattering amplitude with single meson production

$$(2\pi)^4\delta(p_1+p_2-q_1-q_2-k)\,f(p_1p_2|\,q_1q_2|\,k) =$$

$$= \int dx_1 dx_2\, e^{i(p_1-q_1)x_1+i(p_2-q_2)x_2}\, D(x_1-x_2)\int[\delta^4\nu_1]_{-\infty}^{\infty}[\delta^4\nu_2]_{-\infty}^{\infty}$$

$$ig\{e^{ikx_1}\int_{-\infty}^{\infty} d\zeta_1\, e^{-ik[a_1(\zeta_1)+2\int_0^{\zeta_1}\nu_1(\eta)d\eta]} + \tag{2.10}$$

$$+e^{-ikx_2}\int_{-\infty}^{\infty} d\zeta_2\, e^{-ik[a_2(\zeta_2)+2\int_0^{\zeta_2}\nu_2(\eta)d\eta]}\}\times$$

$$\times\int_0^1 d\lambda\,\exp\{i\lambda\chi(x_1-x_2,\nu_1,\nu_2)\}\,.$$

Making in (2.10) the replacement of variables

$$x = x_1 - x_2\ ,\quad y = x_1 + x_2 \tag{2.11}$$

$$\nu_1(\eta)\to\nu_1(\eta)-k\,[\theta(\zeta_1-\eta)-\theta(-\eta)]$$

and integrating over x we obtain

$$f(p_1p_2|\,q_1q_2|\,k)=\int dx\, D(x)\int[\delta^4\nu_1]_{-\infty}^{\infty}[\delta^4\nu_2]_{-\infty}^{\infty}\int_0^1 d\lambda$$

$$\{\int d\zeta_1\exp[-i(p_2-q_2)x-ika_1(\zeta_1)+ik^2|\zeta_1|+i\lambda\kappa_1]+ \tag{2.12}$$

$$+\int d\zeta_2\exp[i(p_1-q_1)x-ika_2(\zeta_2)+ik^2|\zeta_2|+i\lambda\kappa_2]\}\,,$$

where

$$\kappa_1 = g^2\int_{-\infty}^{\infty} d\xi_1\, d\xi_2\, D\{x+a_1(\xi_1)-a_2(\xi_2)+$$

$$+\int_0^{\xi_1}\nu_1(\eta)d\eta-\int_0^{\xi_2}\nu_2(\eta)d\eta+2k[\min(\xi_1,\zeta_1)-\min(0,\xi_1)]\}\,. \tag{2.13}$$

We do not consider the second part of the amplitude which can be obtained by the replacement $q_1 \leftrightarrow q_2$ since in ref./9/ it is shown that in the eikonal region it is nonessential.

Using the method suggested it is not difficult to derive the two-nucleon scattering amplitude with production of two or more meson (see Appendix).

3. Connection between Elastic and Inelastic Scattering Amplitudes and the Eikonal Approximation

In considering the asymptotic behaviour of the scattering amplitude (2.12) we introduce the following notations

$$
\begin{aligned}
t &= T^2 = (p_1 - q_1 - k)^2 = (q_2 - p_2)^2 \\
s &= (p_1 + p_2)^2 .
\end{aligned}
\tag{3.1}
$$

In order to study the inelastic scattering amplitude in the eikonal approximation first of all we consider the problem of factorization of the expression (2.12) at high energies and fixed momentum transfers, namely, we consider the conditions on the components of the meson momentum k under which the elastic and inelastic amplitudes are linked by the following simple relation/14/

$$
f_{inel} = g \left[\frac{1}{2 p_1 k_1 + \mu^2} - \frac{1}{2 q_1 k - \mu^2} + (p_1 \to p_2 , q_1 \to q_2) \right] f_{el}
\tag{3.2}
$$

It is seen from eqs. (2.12) and (2.13) that if it is possible to neglect in the D-function arguments in (2.12) the dependence on the terms containing k compared with the terms containing p_1, p_2, q_1, q_2 then the integrals over ζ_1 and ζ_2 are easily taken and from eq. (2.12) it follows immediately (3.2). This will be valid if the following restrictions on the components of the momentum of a produced meson

$$
k_0 \ll \sqrt{s} , \quad |\vec{k}_\perp| \ll |\vec{T}_\perp| ,
\tag{3.3}
$$

where

$$\vec{k}_\perp = (k_x, k_y),$$

are fulfilled since in c.m.s. $p_{10}, q_{10}, p_{20}, q_{20} \sim \sqrt{s}$ and $p_{1z}, q_{1z}, p_{2z}, q_{2z} \sim \sqrt{s}$.

Now we pass to the conditions which make it possible to obtain for $f_{e\ell}$ in eq. (3.2) an expression in the eikonal form. It is known that in the case of elastic scattering $k=0$ for $s\to\infty$ and t-fixed the transfer vector $T=(p_1-q_1)=(q_2-p_2)$ is perpendicular to the initial momenta and is of the form

$$T=(p_2-p_1)\frac{t}{s}+T_\perp, \qquad \text{where } (T_\perp p_1)=(T_\perp p_2)=0. \qquad (3.4)$$

It is just because of this fact that after having chosen the z-axis as the direction of motion of colliding particles p_1, p_2 in the c.m.s. it is possible /9/ to reduce the integral over d^4x to the integral over the impact parameter $d^2\vec{x}_\perp$ in the expression for the scattering amplitude. In the case under consideration of meson production in the two-nucleon collision the vector is determined by the formula (3.1) and can be expanded in the initial vectors p_1 and p_2 in a form analogous to (3.4):

$$T=(p_2-p_1)\frac{t}{s}-\frac{4(q_1k)}{s}p_2+T_\perp, \qquad (3.5)$$

where

$$(T_\perp p_1)=(T_\perp p_2)=0,\ T_\perp=(0,\vec{T}_\perp,0).$$

Hence it is seen that for $s\to\infty$ and t-fixed the longitudinal component does not vanish since (q_1k) increases with increasing s. However, the term $\frac{(q_1k)}{s}p_2$ may be neglected provided that $\vec{k}_\perp$ and k_z obey the following restrictions

$$\mu, |\vec{k}_\perp| \ll k_z \ll \sqrt{s} \qquad (3.6)$$

since in this case $\frac{(q_1k)}{s}p_{20,z} \sim \frac{\vec{k}_\perp^2+\mu^2}{k_z} \ll 1$.

It is natiral that if the produced meson is a non-relativistic particle

$$|\vec{k}_{\perp}|,\ k_z \ll \mu \tag{3.7}$$

then for the vector T the expression (3.4) holds as well.

Following paper[9] it is possible to obtain the eikonal representation for the elastic part of the amplitude in (3.2) when the conditions (3.3) and (3.6) or (3.7) are satisfied.

Indeed, for the obtained restrictions on the components of the produced meson momentum the expression (2.13) takes the form

$$f(p_1 p_2 | q_1 q_2 | k) = \phi(k) f(p_1 p_2 | q_1 q_2) =$$

$$= 2\phi(k) \int dx\, D(x) \int [\delta^4 \nu_1]_{-\infty}^{\infty} [\delta^4 \nu_2]_{-\infty}^{\infty} \int_0^1 d\lambda\, e^{-ixT} \tag{3.8}$$

$$\exp\{ i g^2 \lambda \int_{-\infty}^{\infty} d\xi_1 d\xi_2\ D[x + a_1(\xi_1) - a_2(\xi_2) + \int_0^{\xi_1} \nu_1(\eta) d\eta - \int_0^{\xi_2} \nu_2(\eta)\ d\eta]\}.$$

The function $f(p_1 p_2 | q_1 q_2)$ coincides with the two-nucleon scattering amplitude for which in ref.[9] the eikonal representation has been found

$$f_{e\ell}^{eik}(p_1 p_2 | q_1 q_2) = \lim_{\epsilon \to 0} - \frac{is}{(2\pi)^4} \int_{|\vec{x}_\perp| \geq \epsilon} d^2\vec{x}_\perp\, e^{i\vec{x}_\perp \vec{T}_\perp} \left(e^{\frac{ig^2}{2\pi s} K_0(\mu |\vec{x}_\perp|)} - 1 \right), \tag{3.9}$$

where
$\vec{T}_\perp^{\,2} \approx t$.

Thus, for the processes with production of mesons with momenta restricted by the conditions (3.3) and (3.6) there exists the following eikonal representation of the amplitude

$$f_{inel}^{eik} = -4\, \frac{3 g \mu^2 k_z^2 - 4 g (kT) k_z^2}{s(k_\perp^2 + \mu^2)^2}\, f_{e\ell}^{eik}(p_1 p_2 | q_1 q_2). \tag{3.10}$$

In conclusion we would like to note that there is another possibility to represent the inelastic process amplitude in the factorized form (3.2), namely in the region of large $kp_{1,2} \sim s$ and $kq_{1,2} \sim s$. In this case the main contribution to the integrals over ζ_1 and ζ_2 will be given by small $\zeta_1 \sim \frac{1}{s}$ and $\zeta_2 \sim \frac{1}{s}$ and therefore the dependence in the argument of the D-functions (2.12) on ζ_1 and ζ_2 may be neglected. However to large values of $kp_{1,2}$ and $kq_{1,2}$ there correspond large values of T^2 for which the eikonal form of the scattering amplitude is invalid.

Taking the opportunity the authors express their deep gratitude to N.N. Bogolubov, V.A. Matveev, A.N. Tavkhelidze for useful discussions and valuable remarks.

Appendix

We write the nucleon scattering amplitude with production of two mesons:

$$f(p_1 p_2 | q_1 q_2 | k_1 k_2) = \int dx\, D(x) \int [\delta^4 \nu_1]_{-\infty}^{\infty} [\delta^4 \nu_2]_{-\infty}^{\infty} \int_0^1 d\lambda \int_{-\infty}^{\infty} d\zeta_1\, d\zeta_2$$

$$\{ \exp[-i(p_2 - q_2 - k_1)x + ik_1^2|\zeta_1| + ik_2^2|\zeta_2| - ik_1 a_1(\zeta_1) - ik_2 a_2(\zeta_2) + i\lambda\Phi_1] +$$

$$+ \exp[i(p_1 - q_1 - k_1)x + ik_1^2|\zeta_2| + ik_2^2|\zeta_1| - ik_1 a_2(\zeta_2) - ik_2 a_1(\zeta_1) + i\lambda\Phi_2] +$$

$$+ \exp[-i(p_2 - q_2)x + ik_1^2|\zeta_1| + ik_2^2|\zeta_2| - ik_1 a_1(\zeta_1) - ik_2 a_1(\zeta_2) +$$

$$+ 2k_1 k_2 \Theta(\zeta_1, \zeta_2) + i\lambda\Phi_3] + \exp[i(p_1 - q_1)x + ik_1^2|\zeta_1| + ik_2^2|\zeta_2| -$$

$$- ik_2 a_2(\zeta_2) - ik_1 a_2(\zeta_1) + 2k_1 k_2 \Theta(\zeta_1, \zeta_2) + i\lambda\Phi_4] \},$$

where

$$\Phi_1 = g^2 \int d\xi_1 d\xi_2 \; D[x + 2\int_0^{\xi_1} \nu_1(\eta) d\eta - 2\int_0^{\xi_2} \nu_2(\eta) d\eta + a_1(\xi_1) - a_2(\xi_2) - K_1]$$

$$K_1 = 2k_1 \tilde{\Theta}(\xi_1, \zeta_1) - 2k_2 \Theta(\xi_2, \zeta_2)$$

$$K_2 = 2k_2 \tilde{\tilde{\Theta}}(\xi_1, \zeta_1) - 2k_1 \Theta(\xi_2, \zeta_2)$$

$$K_3 = 2(k_1 + k_2)\xi_1 \Theta(-\xi_1) + 2k_1 \tilde{\tilde{\Theta}}(\xi_1, \zeta_1) - 2k_2 \Theta(\xi_2, \zeta_1)$$

$$K_4 = K_3 \; (\xi_1 \to \xi_2)$$

$$\tilde{\tilde{\Theta}}(\xi, \zeta) = \min(0, \zeta) - \min(\xi, \zeta)$$

$$\Theta(\zeta_1, \zeta_2) = \min(0, \zeta_1) + \min(0, \zeta_2) + \min(\zeta_1, \zeta_2) \; .$$

References:

1. G. Moliere. Z. Naturforsch., 2A, 133 (1947).
2. R.J. Glauber. "Lectures in Theoretical Physics", vol. 1, p. 315, N.Y., 1959.
3. R. Torgerson. Phys.Rev., 143, 1194 (1966).
4. R. Arnold. Phys.Rev., 153, 1523 (1967).
5. V.R. Garsevanishvili, V.A. Matveev, L.A. Slepchenko, A.N. Tavkhelidze. Phys.Lett ., 29B, 191 (1969). Talk given at the Coral Gables Conference, Miami (1969).
6. V.R. Garsevanishvili, V.A. Matveev, L.A. Slepchenko, A.N. Tavkhelidze,. Miramare-Trieste preprint IC-69-87 (1969).
7. A.A. Logunov, A.N. Tavkhelidze. Nuovo Cim., 29, 380 (1963).
8. H.D.I. Abarbanel, C. Itzykson. Phys.Rev.Lett., 23, 53 (1969).
9. B.M. Barbashov, S.P. Kuleshov, V.A. Matveev, A.N. Sissakian. JINR Preprint, E2-4692 (1969).

10. M. Levy, J. Sucher. Technical Report N. 983, Univ. of Maryland, 1969.

11. И.В.Андреев. ЖЭТФ, 58, 257(1970).

12. Н.Н. Боголюбов, Д.В.Ширков. Введение в теорию квантованных полей, ГИТТЛ, М., 1957.

13. Б.М.Барбашов. ЖЭТФ, 48, 607(1965).

14. В.Н.Грибов. ЯФ, 5, 399 (1967).

Received by Publishing Department
on March 2, 1970.